7.5a Applications from Science and Statistics

work done by a constant force: $W = F \cdot d$

Work done by a variable force: $W = \int_a^b F(x)\,dx$

$\sum F_i \Delta x$


$W = 30$ ft-lb

or

$N \cdot m = J$

Dec 17-7:20 PM

A leaky ~~bucket~~ weighs 22N empty. It is lifted from the ground at a constant rate at a point 20m above the ground by a rope weighing 0.4 N/m. The bucket starts with 70N of water but it leaks at a constant rate and just finishes draining as the bucket reaches the top. Find the amount of work done.

bucket (empty): $W = 22 \cdot 20 = \boxed{440\ J}$

water: F is linear; $x=0$ $F=70$; $x=20$ $F=0$

$m = \frac{70-0}{0-20} = -\frac{7}{2}$

$F = -\frac{7}{2}(x-0) + 70$

$W = \int_0^{20} -\frac{7}{2}x + 70\,dx = \boxed{700\ J}$

rope: 20m; $x=0$ $F = 8 = .4\frac{N}{m} \cdot 20m$; $x=20$ $F=0$

$F = -.4x + 8$

$W = \int_0^{20} -.4x + 8\,dx = \boxed{80\ J}$

$Total = 440 + 700 + 80 = 1220\ J$

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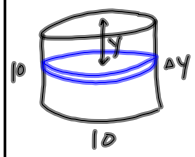
How much work does it take to pump all the water over the rim of a cylindrical tank of height 10ft and diameter 10ft?




$\Delta V = \pi r^2 \Delta y = \pi \cdot 5^2 \cdot \Delta y$

$\Delta F = 62.4\frac{lb}{ft^3} \cdot \pi \cdot 5^2 \Delta y\ ft^3$ (Force (weight))

$\Delta W = 62.4\,\pi \cdot 5^2 \cdot y\,\Delta y$

$W = \int_0^{10} 62.4\,\pi\,25\,y\,dy$

$= 245{,}044\ J = 245\ kJ$

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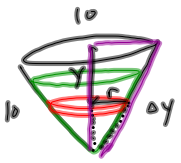
A conical tank of height and diameter 10ft is filled to within 2 ft of the top with ~~olive oil~~ weighing 57 lb/ft^3. How much work does it take to pump the oil to the rim of the tank?




$\Delta V = \pi r^2 \Delta y$

$\Delta F = 57\,\pi r^2 \Delta y$

$\Delta W = 57\,\pi r^2 y\,\Delta y$

need r in terms of y

$\frac{5}{10} = \frac{r}{10-y}$

$r = \frac{10-y}{2}$

$W = \int_2^{10} 57\,\pi \left(\frac{10-y}{2}\right)^2 y\,dy$

$= 9728\,\pi < 30{,}561.4$ ft-lb

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